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Use of Mode Localization in Passive Control of Structural Buckling

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I. Introduction

ANDERSON localization, a phenomenon that was observed first in the field of solid state physics, has recently been given a lot of attention by structural dynamicists. Although the presence of irregularities in periodic lattices led to the localization of the electron eigenstates, the motion of a disordered elastic system was found to be confined to a limited part of the system. This study is an endeavor to introduce the mode localization phenomenon into the area of structural stability. By means of inducing deliberately some irregularities in the system, we may confine the structural buckling to only a limited part of the system. This process might be looked at as a passive control of buckling.

To cover the relevant literature in the area, we mention that the phenomenon of electron field localization in solid state physics[1] was extended to the field of structural dynamics by Hodge[2] and Hodge and Woodhouse.[3] Since then, the problem of mode localization has stimulated a number of structural dynamicists. Vibration localization in discrete mechanical systems has been considered by Pierre and Dowell,[4] Bendiksen,[5] Pierre,[6,7] and Wei and Pierre.[8] Vibration confinement in multispan beams has been considered by Pierre et al.,[9] Bouzit and Pierre,[10] and Lust et al.[11] Localization of normal modes of nonlinear systems has been recently considered by Vakakis.[12]

The applicability of the localization phenomenon to structural buckling was only addressed by Pierre and Plaut,[13] who examined the occurrence of buckling pattern localization in a two-span disordered column.

In the present Note, the effect of mode localization in a nearly periodic multispan column is studied using an exact formulation with the aid of the transfer matrix method. Mode localization is achieved by imposing specific types of irregularities, such as the inclusion of intermediate torsional springs and/or displacing intermediate supports. The buckled mode shapes indicate quite a strong confinement of the buckling to a fraction of the column.

II. Problem Formulation

We consider a multispan column of length $L\ (=\Sigma_{j=1}^{N}\ell_j)$, where the index j refers to a single span and N is the total number of spans. The panel is simply supported at both ends of each span. Moreover, identical torsional springs, each of stiffness c, are placed at the interior supports which exert restoring moments at these locations.

Received Aug. 26, 1993; revision received Feb. 15, 1994; accepted for publication Feb. 16, 1994; presented as Paper 94-1584 at the AIAA/ASME/ASCE/AHS/ASC 35th Structures, Structural Dynamics, and Materials Conference, Hilton Head, SC, April 18–20, 1994.

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The governing equation of the displacement of the individual panel is

$$\frac{d^4 w_j}{dx_j^4} + k^2 \frac{d^2 w_j}{dx_j^2} = 0, \quad \text{in} \quad 0 < x_j < \ell_j \tag{1}$$

where w_j is the transverse displacement, $k^2 = P/EI$, P is the buckling axial load, and EI is the bending stiffness of the material. The general solution of Eq. (1) is

$$w_j = A_j \sin(kx_j) + B_j \cos(kx_j) + C_j x_j + D_j \tag{2}$$

where A_j, B_j, C_j, and D_j are arbitrary constants.

The boundary conditions at the ends of the panel element are

$$w_j(0) = w_j(\ell_j) = 0 \tag{3}$$

$$\frac{dw_j}{dx_j}(0) = \theta_{L_j}, \qquad \frac{dw_j}{dx_j}(\ell_j) = \theta_{R_j} \tag{4}$$

$$\begin{aligned} EI\frac{d^2 w_j}{dx_j^2}(0) + (1/2)c\theta_{L_j} &= \hat{M}_{L_j} \\ -EI\frac{d^2 w_j}{dx_j^2}(\ell_j) + (1/2)c\theta_{R_j} &= \hat{M}_{R_j} \end{aligned} \tag{5}$$

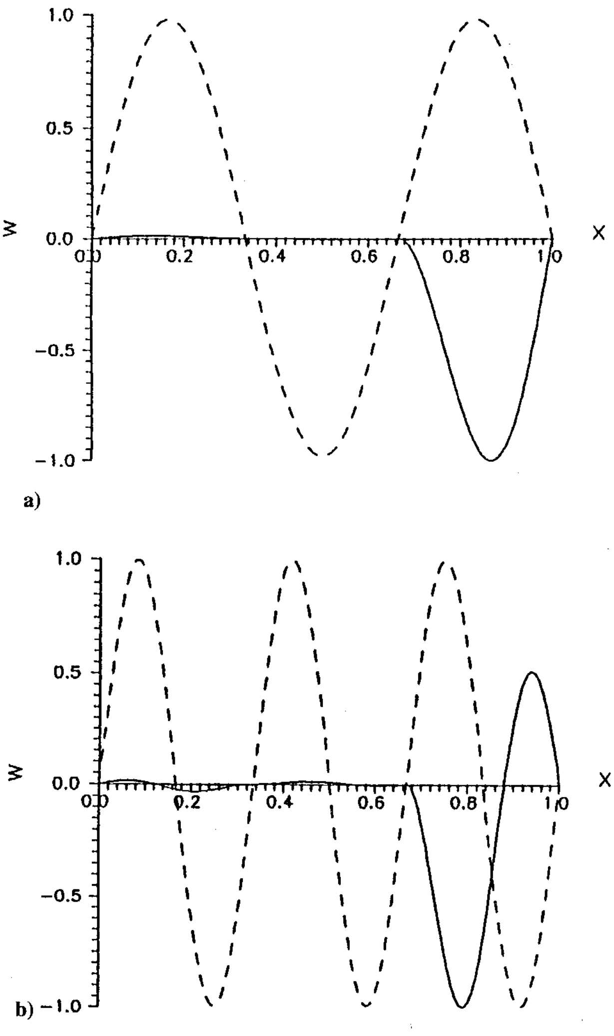

Fig. 1 Three-span column: a) first mode shape and b) second mode shape; ordered (dashed line) and disordered (solid line).

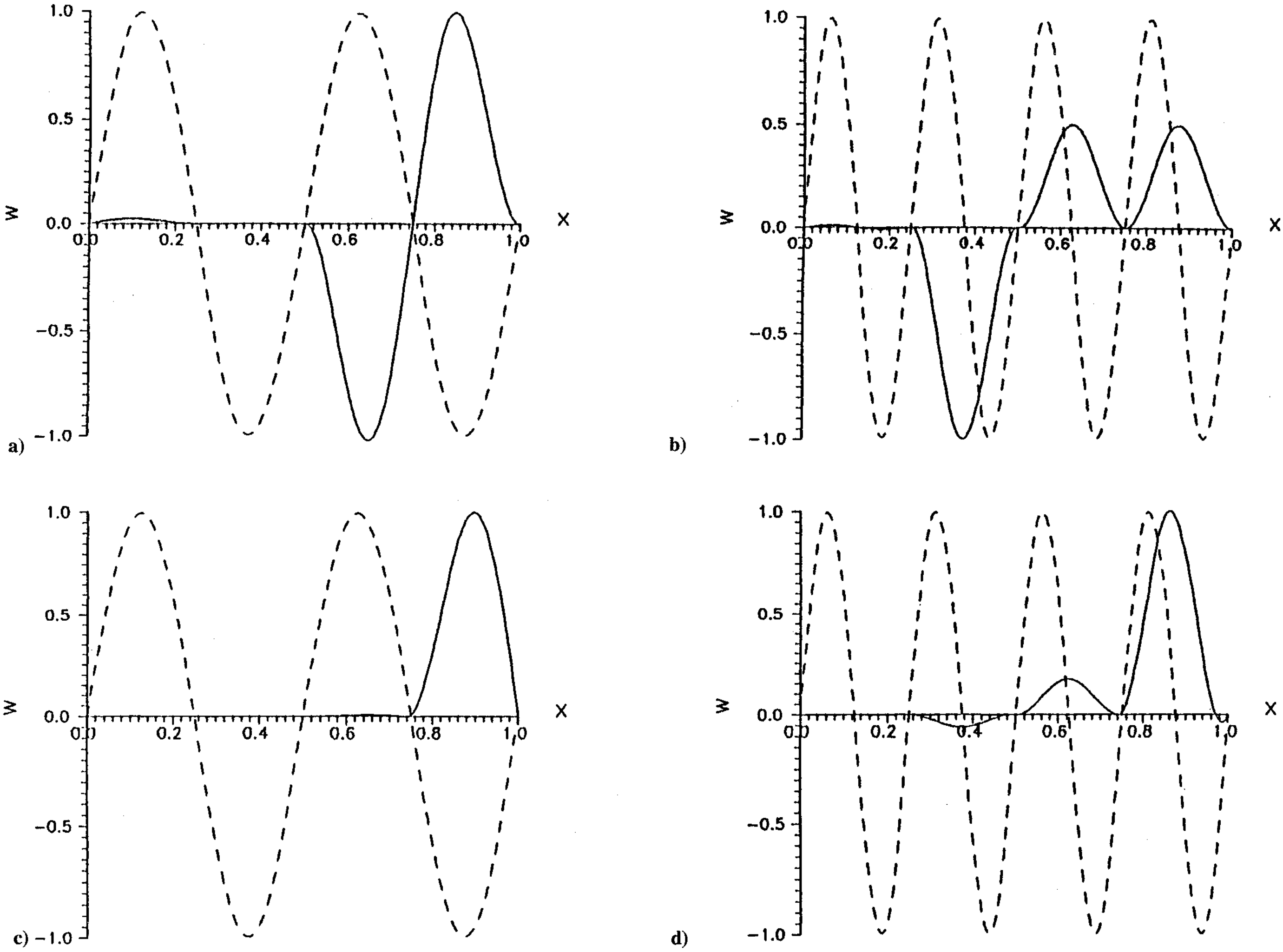

Fig. 2 Four-span column: a) first mode shape, b) second mode shape, c) first mode shape, and d) second mode shape; ordered (dashed line) and disordered (solid line).

To build the element stiffness matrix, we substitute the solution (2) into the boundary conditions (3–5), eliminate the arbitrary coefficients, and obtain

$$\begin{Bmatrix} M_{L_j} \\ M_{R_j} \end{Bmatrix} = \begin{bmatrix} S_{11j} & S_{12j} \\ S_{21j} & S_{22j} \end{bmatrix} \begin{Bmatrix} \theta_{L_j} \\ \theta_{R_j} \end{Bmatrix} \tag{6}$$

where

$$M_{L,R_j} = \frac{\hat{M}_{L,R_j}}{EI} \tag{7}$$

$$S_{11j} = S_{22j} = -\frac{k^3\ell_j\cosh(k\ell_j) - k^2\sin(k\ell_j)}{k^2\ell_j\sin(k\ell_j) + 2k[\cos(k\ell_j) - 1]} + \frac{c}{2EI} \tag{8}$$

$$S_{12j} = S_{21j} = \frac{k^3\ell_j - k^2\sin(k\ell_j)}{k^2\ell_j\sin(k\ell_j) + 2k[\cos(k\ell_j) - 1]} \tag{9}$$

The exact stiffness matrix in Eq. (6) can be utilized to derive the transfer matrix that relates the state variable at one end of the column to those at the other as

$$\begin{Bmatrix} \theta_{R_j} \\ M_{R_j} \end{Bmatrix} = \begin{bmatrix} t_{11j} & t_{12j} \\ t_{21j} & t_{22j} \end{bmatrix} \begin{Bmatrix} \theta_{L_j} \\ M_{L_j} \end{Bmatrix} \tag{10}$$

where

$$t_{11j} = -\frac{S_{11j}}{S_{12j}}, \quad t_{12j} = \frac{1}{S_{12j}}$$
$$t_{21j} = -\frac{S_{12j}S_{21j} - S_{11j}S_{22j}}{S_{12j}}, \qquad t_{22j} = \frac{S_{22j}}{S_{12j}} \tag{11}$$

Starting from the transfer matrix for the span j, we can find the transfer matrix for the total column by relating the field variables at the right end of the column to those at the left end of the column by invoking the continuity condition of the state variables at the intermediate supports. Hence, we can relate the slope and moment at the right end of the total column to those at its left through the product

$$[T] = [t_n][t_{n-1}]\ldots[t_2][t_1] \tag{12}$$

The result is

$$\begin{Bmatrix} \theta_{R_N} \\ M_{R_N} \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} \theta_{L_1} \\ M_{L_1} \end{Bmatrix} \tag{13}$$

The characteristic equation of the total column is found by applying the boundary conditions at both ends of the column. The result is

$$T_{21} = 0 \tag{14}$$

This equation is solved numerically for the buckling loads of the column. The mode shapes are then found by a backward solution of Eqs. (2–14).

III. Numerical Examples

In this section, we present the mode shapes for panels with different types of irregularities and compare the results with those for the ordered ones.

A. Three-Span Column

We consider a column consisting of three segments of equal length. The column in this case is said to be ordered. When torsional springs each with $c/EI = 100$ are placed at the intermediate supports, they exert restoring moments and hence weaken the coupling of the three spans. The column in this case is disordered.

The effect of imposing the springs on the buckling load is favorable: $k_1 = 0.785$ and $k_2 = 1.571$ for the ordered case, whereas $k_1 = 1.127$ and $k_2 = 1.938$ for the disordered case. Clearly both the first and second buckling loads are increased.

The shapes of the first two modes of both the ordered and disordered columns are shown in Figs. 1a and 1b. We note that the magnitudes of the deflection are the same in each span in the ordered case, and the magnitude of the deflection in one span is much larger than the magnitude of the deflection in the other two in the disordered case.

These results are quite encouraging. The disorder does not only localize the buckling over one-third of the column but also increases the first buckling load by 43% and the second buckling load by 23%.

B. Four-Span Column

An ordered equispaced supported column is considered. The column is disordered by placing three identical springs each with $c/EI = 100$ at the interior supports. In Figs. 2a and 2b we show the first and the second mode shapes, respectively. Clearly, the first mode is localized over two spans rather than over one span because the two middle spans are the same and the left and right half of the system are disordered. Also, the second mode is weakly localized; buckling disappeared over a single span only.

The disorder increases the buckling loads: $k_1 = 1.047$ and $k_2 = 2.094$ for the ordered column, whereas $k_1 = 1.502$ and $k_2 = 2.116$ for the disordered column. The first buckling load is increased by 43% and the second buckling mode is increased by 2%.

To localize the modes of the four-span beam further, we introduce an additional disorder by displacing one of the interior supports such that $\ell_1/\ell_2 = 1.0$ and $\ell_3/\ell_4 = 0.935$. A stronger localization is achieved. The fundamental mode is localized entirely over only one span, as shown in Fig. 2c, and the second mode is localized globally over one span, as shown in Fig. 2d.

The influence of the geometric irregularity on the buckling loads is to decrease the first buckling mode where k_1 becomes 1.478, and to increase the second buckling load where k_1 becomes 2.152.

IV. Conclusion

The buckling problem of nearly periodic structures has been treated. Imposing certain types of irregularities, including intermediate torsional springs and displaced intermediate supports, confines the buckling to a part of the structure. The effect of such irregularities on the buckling loads has been generally favorable. The results show that the concept of mode localization can be used to passively control the structural stability.

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